1998 AP® CALCULUS AB EXAMINATION
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DO NOT WRITE IN THIS AREA.

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ETS USE ONLY

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DO NOT WRITE IN THIS AREA.
CALCULUS AB

A CALCULATOR CANNOT BE USED ON PART A OF SECTION I. A GRAPHLING CALCULATOR FROM THE
APPRVED LIST IS REQUIRED ON PART B OF SECTION I AND FOR SECTION II OF THE EXAMINATION.
CALCULATOR MEMORIES NEED NOT BE CLEARED. COMPUTERS, NONGRAPHING SCIENTIFIC
CALCULATORS, CALCULATORS WITH QWERTY KEYBOARDS, AND ELECTRONIC WRITING PADS ARE
NOT ALLOWED. CALCULATORS MAY NOT BE SHARED AND COMMUNICATION BETWEEN CALCULATORS
IS PROHIBITED DURING THE EXAMINATION. ATTEMPTS TO REMOVE TEST MATERIALS FROM THE ROOM
BY ANY METHOD WILL RESULT IN THE INVALIDATION OF TEST SCORES.

SECTION I

Time — 1 hour and 45 minutes
All questions are given equal weight.
Percent of total grade — 50

Part A: 55 minutes, 28 multiple-choice questions
A calculator is NOT allowed.

Part B: 50 minutes, 17 multiple-choice questions
A graphing calculator is required.

Parts A and B of Section I are printed in this examination booklet; Section II, which consists of longer problems, is printed
in a separate booklet.

General Instructions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.

INDICATE YOUR ANSWERS TO QUESTIONS IN PART A ON PAGE 2 OF THE SEPARATE ANSWER SHEET. THE
ANSWERS TO QUESTIONS IN PART B SHOULD BE INDICATED ON PAGE 3 OF THE ANSWER SHEET. No credit
will be given for anything written in this examination booklet, but you may use the booklet for notes or scratchwork. After
you have decided which of the suggested answers is best, COMPLETELY fill in the corresponding oval on the answer sheet.
Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Example.

What is the arithmetic mean of the numbers 1, 3, and 6?

(A) 1

(B) \( \frac{7}{3} \)

(C) 3

(D) \( \frac{10}{3} \)

(E) \( \frac{7}{2} \)

Sample Answer

A → B → C → E

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. In this section
of the examination, as a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly
will be subtracted from the number of questions you answer correctly. It is improbable, therefore, that mere guessing will
improve your score significantly; it may even lower your score, and it does take time. If, however, you are not sure of the
best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices as wrong,
your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

Use your time effectively, working as rapidly as you can without losing accuracy. Do not spend too much time on questions
that are too difficult. Go on to other questions and come back to the difficult ones later if you have time. It is not expected
that everyone will be able to answer all the multiple-choice questions.

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Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test booklet. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function \( f \) is assumed to be the set of all real numbers \( x \) for which \( f(x) \) is a real number.

1. What is the \( x \)-coordinate of the point of inflection on the graph of \( y = \frac{1}{3}x^3 + 5x^2 + 24 \)?

   (A) 5  (B) 0  (C) \(-\frac{10}{3}\)  (D) -5  (E) -10
2. The graph of a piecewise-linear function $f$, for $-1 \leq x \leq 4$, is shown above. What is the value of 
\[ \int_{-1}^{4} f(x) \, dx \]?

(A) 1  
(B) 2.5  
(C) 4  
(D) 5.5  
(E) 8

3. \[ \int_{1}^{2} \frac{1}{x^2} \, dx = \]

(A) $-\frac{1}{2}$  
(B) $\frac{7}{24}$  
(C) $\frac{1}{2}$  
(D) 1  
(E) $2 \ln 2$
4. If \( f \) is continuous for \( a \leq x \leq b \) and differentiable for \( a < x < b \), which of the following could be false?

(A) \( f'(c) = \frac{f(b) - f(a)}{b - a} \) for some \( c \) such that \( a < c < b \).

(B) \( f'(c) = 0 \) for some \( c \) such that \( a < c < b \).

(C) \( f \) has a minimum value on \( a \leq x \leq b \).

(D) \( f \) has a maximum value on \( a \leq x \leq b \).

(E) \( \int_a^b f(x) \, dx \) exists.

5. \( \int_0^\pi \sin t \, dt = \)

(A) \( \sin x \)  \hspace{1cm} (B) \( -\cos x \)  \hspace{1cm} (C) \( \cos x \)  \hspace{1cm} (D) \( \cos x - 1 \)  \hspace{1cm} (E) \( 1 - \cos x \)
6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $-\frac{7}{2}$  (B) $-2$  (C) $\frac{2}{7}$  (D) $\frac{3}{2}$  (E) $\frac{7}{2}$

7. $\int_{1}^{e} \left( \frac{x^2 - 1}{x} \right) \, dx =$

(A) $e - \frac{1}{e}$  (B) $e^2 - 2$  (C) $\frac{e^2}{2} - e + \frac{1}{2}$  (D) $e^2 - 2$  (E) $\frac{e^2}{2} - \frac{3}{2}$
8. Let \( f \) and \( g \) be differentiable functions with the following properties:
   (i) \( g(x) > 0 \) for all \( x \)
   (ii) \( f(0) = 1 \)
If \( h(x) = f(x)g(x) \) and \( h'(x) = f(x)g'(x) \), then \( f(x) = \)

(A) \( f'(x) \)  (B) \( g(x) \)  (C) \( e^x \)  (D) 0  (E) 1

9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500  (B) 600  (C) 2,400  (D) 3,000  (E) 4,800
10. What is the instantaneous rate of change at $x = 2$ of the function $f$ given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) $-2$  (B) $\frac{1}{6}$  (C) $\frac{1}{2}$  (D) $2$  (E) $6$

11. If $f$ is a linear function and $0 < a < b$, then $\int_{a}^{b} f''(x) \, dx =$

(A) $0$  (B) $1$  (C) $\frac{ab}{2}$  (D) $b - a$  (E) $\frac{b^2 - a^2}{2}$
12. If \( f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4 \end{cases} \), then \( \lim_{x \to 2} f(x) \) is

(A) \( \ln 2 \)   (B) \( \ln 8 \)   (C) \( \ln 16 \)   (D) 4   (E) nonexistent

13. The graph of the function \( f \) shown in the figure above has a vertical tangent at the point \((2, 0)\) and horizontal tangents at the points \((1, -1)\) and \((3, 1)\). For what values of \( x \), \(-2 < x < 4\), is \( f \) not differentiable?

(A) 0 only   (B) 0 and 2 only   (C) 1 and 3 only   (D) 0, 1, and 3 only   (E) 0, 1, 2, and 3
14. A particle moves along the x-axis so that its position at time \( t \) is given by \( x(t) = t^2 - 6t + 5 \). For what value of \( t \) is the velocity of the particle zero?

(A) 1  
(B) 2  
(C) 3  
(D) 4  
(E) 5

15. If \( F(x) = \int_0^x \sqrt{t^3 + 1} \, dt \), then \( F'(2) = \)

(A) \(-3\)  
(B) \(-2\)  
(C) 2  
(D) 3  
(E) 18
16. If \( f(x) = \sin (e^{-x}) \), then \( f'(x) = \)

(A) \(-\cos(e^{-x})\)
(B) \(\cos(e^{-x}) + e^{-x}\)
(C) \(\cos(e^{-x}) - e^{-x}\)
(D) \(e^{-x} \cos(e^{-x})\)
(E) \(-e^{-x} \cos(e^{-x})\)

17. The graph of a twice-differentiable function \( f \) is shown in the figure above. Which of the following is true?

(A) \( f(1) < f'(1) < f''(1) \)
(B) \( f(1) < f''(1) < f'(1) \)
(C) \( f'(1) < f(1) < f''(1) \)
(D) \( f''(1) < f(1) < f'(1) \)
(E) \( f''(1) < f'(1) < f(1) \)
18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

(A) $y = 2x + 1$  (B) $y = x + 1$  (C) $y = x$  (D) $y = x - 1$  (E) $y = 0$

19. If $f''(x) = x(x + 1)(x - 2)^2$, then the graph of $f$ has inflection points when $x =$

(A) $-1$ only  (B) $2$ only  (C) $-1$ and $0$ only  (D) $-1$ and $2$ only  (E) $-1$, $0$, and $2$ only
20. What are all values of $k$ for which $\int_{-3}^{k} x^2 \, dx = 0$?

(A) $-3$  (B) $0$  (C) $3$  (D) $-3$ and $3$  (E) $-3, 0, \text{ and } 3$

21. If $\frac{dy}{dt} = ky$ and $k$ is a nonzero constant, then $y$ could be

(A) $2e^{kty}$  (B) $2e^{kt}$  (C) $e^{kt} + 3$  (D) $kty + 5$  (E) $\frac{1}{2} ky^2 + \frac{1}{2}$
22. The function \( f \) is given by \( f(x) = x^4 + x^2 - 2 \). On which of the following intervals is \( f \) increasing?

(A) \( \left( -\frac{1}{\sqrt{2}}, \infty \right) \)

(B) \( \left( -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \)

(C) \( (0, \infty) \)

(D) \( (-\infty, 0) \)

(E) \( (-\infty, -\frac{1}{\sqrt{2}}) \)
23. The graph of $f$ is shown in the figure above. Which of the following could be the graph of the derivative of $f$?

(A) 

(B) 

(C) 

(D) 

(E)
24. The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^8 - 3t^3 + 12t + 4$ is

(A) 9  (B) 12  (C) 14  (D) 21  (E) 40

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$  (B) $\frac{8}{3}$  (C) 4  (D) $\frac{14}{3}$  (E) $\frac{16}{3}$
26. The function \( f \) is continuous on the closed interval \([0, 2]\) and has values that are given in the table above. The equation \( f(x) = \frac{1}{2} \) must have at least two solutions in the interval \([0, 2]\) if \( k = \)

(A) 0  (B) \( \frac{1}{2} \)  (C) 1  (D) 2  (E) 3

27. What is the average value of \( y = x^2 \sqrt{x^3 + 1} \) on the interval \([0, 2]\) ?

(A) \( \frac{26}{9} \)  (B) \( \frac{52}{9} \)  (C) \( \frac{26}{3} \)  (D) \( \frac{52}{3} \)  (E) 24
28. If \( f(x) = \tan(2x) \), then \( f'\left(\frac{\pi}{6}\right) = \)

(A) \( \sqrt{3} \)  
(B) \( 2\sqrt{3} \)  
(C) 4  
(D) \( 4\sqrt{3} \)  
(E) 8
CALCULUS AB
SECTION I. Part B
Time — 50 minutes
Number of questions — 17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After exam-
ing the form of the choices, decide which is the best of the choices given and fill in the corresponding
oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too
much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS
TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given.
When this happens, select from among the choices the number that best approximates the exact
numerical value.

(2) Unless otherwise specified, the domain of a function \( f \) is assumed to be the set of all real numbers \( x \)
for which \( f(x) \) is a real number.
76. The graph of a function \( f \) is shown above. Which of the following statements about \( f \) is false?

(A) \( f \) is continuous at \( x = a \).

(B) \( f \) has a relative maximum at \( x = a \).

(C) \( x = a \) is in the domain of \( f \).

(D) \( \lim_{x \to a^+} f(x) \) is equal to \( \lim_{x \to a^-} f(x) \).

(E) \( \lim_{x \to a} f(x) \) exists.
77. Let \( f \) be the function given by \( f(x) = 3e^{2x} \) and let \( g \) be the function given by \( g(x) = 6x^3 \). At what value of \( x \) do the graphs of \( f \) and \( g \) have parallel tangent lines?

(A) \(-0.701\)
(B) \(-0.567\)
(C) \(-0.391\)
(D) \(-0.302\)
(E) \(-0.258\)

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference \( C \), what is the rate of change of the area of the circle, in square centimeters per second?

(A) \(-(0.2)\pi C\)
(B) \(-(0.1)C\)
(C) \(-\frac{(0.1)C}{2\pi}\)
(D) \((0.1)^2C\)
(E) \((0.1)^2\pi C\)
79. The graphs of the derivatives of the functions $f$, $g$, and $h$ are shown above. Which of the functions $f$, $g$, or $h$ have a relative maximum on the open interval $a < x < b$?

(A) $f$ only  
(B) $g$ only  
(C) $h$ only  
(D) $f$ and $g$ only  
(E) $f$, $g$, and $h$  

80. The first derivative of the function $f$ is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does $f$ have on the open interval $(0, 10)$?

(A) One  
(B) Three  
(C) Four  
(D) Five  
(E) Seven
81. Let \( f \) be the function given by \( f(x) = |x| \). Which of the following statements about \( f \) are true?

I. \( f \) is continuous at \( x = 0 \).
II. \( f \) is differentiable at \( x = 0 \).
III. \( f \) has an absolute minimum at \( x = 0 \).

(A) I only  (B) II only  (C) III only  (D) I and III only  (E) II and III only

82. If \( f \) is a continuous function and \( F'(x) = f(x) \) for all real numbers \( x \), then \( \int_{1}^{3} f(2x)dx = \)

(A) \( 2F(3) - 2F(1) \)
(B) \( \frac{1}{2} F(3) - \frac{1}{2} F(1) \)
(C) \( 2F(6) - 2F(2) \)
(D) \( F(6) - F(2) \)
(E) \( \frac{1}{2} F(6) - \frac{1}{2} F(2) \)
83. If \( a \neq 0 \), then \( \lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4} \) is

(A) \( \frac{1}{a^2} \)  (B) \( \frac{1}{2a^2} \)  (C) \( \frac{1}{6a^2} \)  (D) 0  (E) nonexistent

84. Population \( y \) grows according to the equation \( \frac{dy}{dt} = ky \), where \( k \) is a constant and \( t \) is measured in years. If the population doubles every 10 years, then the value of \( k \) is

(A) 0.069  (B) 0.200  (C) 0.301  (D) 3.322  (E) 5.000
85. The function \( f \) is continuous on the closed interval \([2, 8]\) and has values that are given in the table above. Using the subintervals \([2, 5]\), \([5, 7]\), and \([7, 8]\), what is the trapezoidal approximation of 
\[
\int_{2}^{8} f(x) \, dx 
\]
(A) 110  (B) 130  (C) 160  (D) 190  (E) 210

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86. The base of a solid is a region in the first quadrant bounded by the \( x \)-axis, the \( y \)-axis, and the line \( x + 2y = 8 \), as shown in the figure above. If cross sections of the solid perpendicular to the \( x \)-axis are semicircles, what is the volume of the solid?

(A) 12.566  (B) 14.661  (C) 16.755  (D) 67.021  (E) 134.041
87. Which of the following is an equation of the line tangent to the graph of \( f(x) = x^4 + 2x^2 \) at the point where \( f'(x) = 1 \)?

(A) \( y = 8x - 5 \)  
(B) \( y = x + 7 \)  
(C) \( y = x + 0.763 \)  
(D) \( y = x - 0.122 \)  
(E) \( y = x - 2.146 \)

88. Let \( F(x) \) be an antiderivative of \( \frac{(\ln x)^3}{x} \). If \( F(1) = 0 \), then \( F(9) = \)

(A) 0.048  
(B) 0.144  
(C) 5.827  
(D) 23.308  
(E) 1,640.250
89. If \( g \) is a differentiable function such that \( g(x) < 0 \) for all real numbers \( x \) and if \( f'(x) = (x^2 - 4)g(x) \), which of the following is true?

(A) \( f \) has a relative maximum at \( x = -2 \) and a relative minimum at \( x = 2 \).
(B) \( f \) has a relative minimum at \( x = -2 \) and a relative maximum at \( x = 2 \).
(C) \( f \) has relative minima at \( x = -2 \) and at \( x = 2 \).
(D) \( f \) has relative maxima at \( x = -2 \) and at \( x = 2 \).
(E) It cannot be determined if \( f \) has any relative extrema.

90. If the base \( b \) of a triangle is increasing at a rate of 3 inches per minute while its height \( h \) is decreasing at a rate of 3 inches per minute, which of the following must be true about the area \( A \) of the triangle?

(A) \( A \) is always increasing.
(B) \( A \) is always decreasing.
(C) \( A \) is decreasing only when \( b < h \).
(D) \( A \) is decreasing only when \( b > h \).
(E) \( A \) remains constant.
91. Let \( f \) be a function that is differentiable on the open interval \((1, 10)\). If \( f(2) = -5 \), \( f(5) = 5 \), and \( f(9) = -5 \), which of the following must be true?

   I. \( f \) has at least 2 zeros.
   II. The graph of \( f \) has at least one horizontal tangent.
   III. For some \( c, 2 < c < 5 \), \( f(c) = 3 \).

(A) None  
(B) I only  
(C) I and II only  
(D) I and III only  
(E) I, II and III

92. If \( 0 \leq k < \frac{\pi}{2} \) and the area under the curve \( y = \cos x \) from \( x = k \) to \( x = \frac{\pi}{2} \) is 0.1, then \( k = \)

(A) 1.471  
(B) 1.414  
(C) 1.277  
(D) 1.120  
(E) 0.436

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY.
CHECK YOUR WORK ON PART B ONLY.
DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE PLACED YOUR AP NUMBER LABEL ON YOUR
ANSWER SHEET AND HAVE WRITTEN AND GRIDDED YOUR AP NUMBER
IN THE APPROPRIATE SECTION OF YOUR ANSWER SHEET.

AFTER TIME HAS BEEN CALLED, ANSWER QUESTIONS 93-96.
93. Which graphing calculator did you use during the examination?

(A) Casio 6300, Casio 7000, Casio 7300, Casio 7400, or Casio 7700
(B) Texas Instruments TI-80 or TI-81
(C) Casio 9700, Casio 9800, Casio 9850, Sharp 9200, Sharp 9300, Texas Instruments TI-82, Texas Instruments TI-83, Texas Instruments TI-85, or Texas Instruments TI-86
(D) Hewlett-Packard HP-48 series or HP-38G
(E) Some other calculator

94. During your Calculus AB course, which of the following best describes your calculator use?

(A) I used my own graphing calculator.
(B) I used a graphing calculator furnished by my school, both in class and at home.
(C) I used a graphing calculator furnished by my school only in class.
(D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
(E) I did not use a graphing calculator.

95. During your Calculus AB course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?

(A) Almost every class
(B) About three-quarters of the classes
(C) About one-half of the classes
(D) About one-quarter of the classes
(E) Seldom or never

96. During your Calculus AB course, which of the following describes approximately how often you were allowed to use a graphing calculator on tests?

(A) Almost all of the time
(B) About three-quarters of the time
(C) About one-half of the time
(D) About one-quarter of the time
(E) Seldom or never
CALCULUS AB
SECTION II
Time — 1 hour and 30 minutes
Number of problems — 6
Percent of total grade — 50

GENERAL INSTRUCTIONS
You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight. The problems are printed in the booklet and in the green insert; it may be easier for you to first look over all problems in the insert. When you are told to begin, open your booklet, carefully tear out the green insert, and start to work.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

• You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.

• Show all your work. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive credit.

• Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

• You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

• Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, \( \int_1^5 x^2 \, dx \) may not be written as lnln(x^2, x, 1, 5).

• Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

• Unless otherwise specified, the domain of a function \( f \) is assumed to be the set of all real numbers \( x \) for which \( f(x) \) is a real number.
CALCULUS AB
SECTION II
Time — 1 hour and 30 minutes
Number of problems — 6
Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

General instructions for this section are printed on the back cover of this booklet.

1. Let $R$ be the region bounded by the $x$-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

(a) Find the area of the region $R$.

(b) Find the value of $h$ such that the vertical line $x = h$ divides the region $R$ into two regions of equal area.
(c) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

(d) The vertical line $x = k$ divides the region $R$ into two regions such that when these two regions are revolved about the $x$-axis, they generate solids with equal volumes. Find the value of $k$. 
2. Let $f$ be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of $f$. Justify that your answer is an absolute minimum.
(c) What is the range of \( f \)?

(d) Consider the family of functions defined by \( y = bxe^{bx} \), where \( b \) is a nonzero constant. Show that the absolute minimum value of \( bxe^{bx} \) is the same for all nonzero values of \( b \).
3. The graph of the velocity \( v(t) \), in ft/sec, of a car traveling on a straight road, for \( 0 \leq t \leq 50 \), is shown above. A table of values for \( v(t) \), at 5 second intervals of time \( t \), is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

(b) Find the average acceleration of the car, in ft/sec\(^2\), over the interval \( 0 \leq t \leq 50 \).
(c) Find one approximation for the acceleration of the car, in ft/sec², at \( t = 40 \). Show the computations you used to arrive at your answer.

(d) Approximate \( \int_{0}^{50} v(t) \, dt \) with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.
4. Let $f$ be a function with $f(1) = 4$ such that for all points $(x, y)$ on the graph of $f$ the slope is given by $\frac{3x^2 + 1}{2y}$.

(a) Find the slope of the graph of $f$ at the point where $x = 1$.

(b) Write an equation for the line tangent to the graph of $f$ at $x = 1$ and use it to approximate $f(1.2)$. 
(c) Find \( f(x) \) by solving the separable differential equation \( \frac{dy}{dx} = \frac{3x^2 + 1}{2y} \) with the initial condition \( f(1) = 4 \).

(d) Use your solution from part (c) to find \( f(1.2) \).
5. The temperature outside a house during a 24-hour period is given by

\[ F(t) = 80 - 10 \cos \left( \frac{\pi t}{12} \right), \quad 0 \leq t \leq 24, \]

where \( F(t) \) is measured in degrees Fahrenheit and \( t \) is measured in hours.

(a) Sketch the graph of \( F \) on the grid below.

(b) Find the average temperature, to the nearest degree Fahrenheit, between \( t = 6 \) and \( t = 14 \).
(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of $t$ was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of $0.05$ per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?
6. Consider the curve defined by $2y^2 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

(b) Write an equation of each horizontal tangent line to the curve.
(c) The line through the origin with slope $-1$ is tangent to the curve at point $P$. Find the $x$- and $y$-coordinates of point $P$. 

END OF EXAMINATION